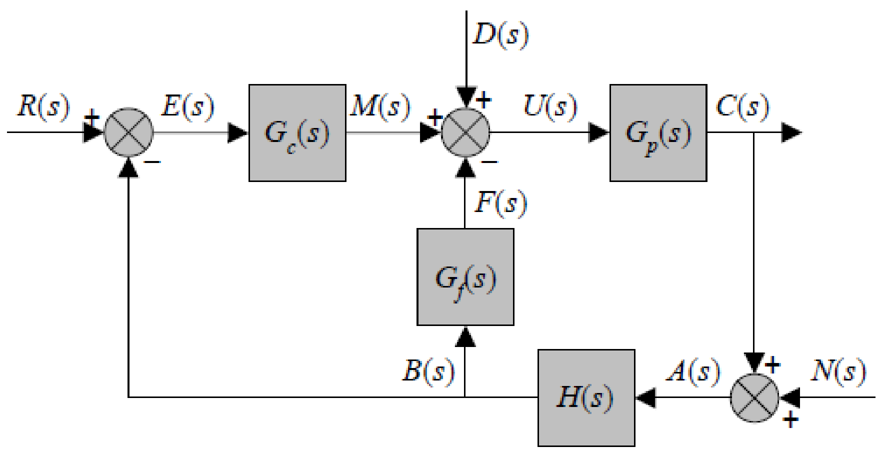
CITY COLLEGE

CITY UNIVERSITY OF NEW YORK

HOMEWORK #3



TWO –DOF CONTROL SYSTEM WITH FEEDBACK COMPENSATION

ME 411: System Modeling Analysis and Control

Fall 2010

Prof. B. Liaw

Submitted By: Pradip Thapa

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1.0 Nomenclature

$G_p(s) = transferfucntion\ of\ the\ plant$

$G_c(s) = transferfucntion\ of\ the\ controller$

$G_f(s) = transferfucntion\ of\ the\ feedback\ compensator$

$H(s) = transferfucntion\ of\ the\ feedback\ element$

$C(s) = transferfucntion\ of\ the\ control\ output\ c(t)$

$E(s) = transferfucntion\ of\ the\ error\ or\ actuating\ signal\ e(t)$

$R(s) = transferfucntion\ of\ the\ reference\ input\ r(t)$

$D(s) = transferfucntion\ of\ the\ disturbance\ input\ d(t)$

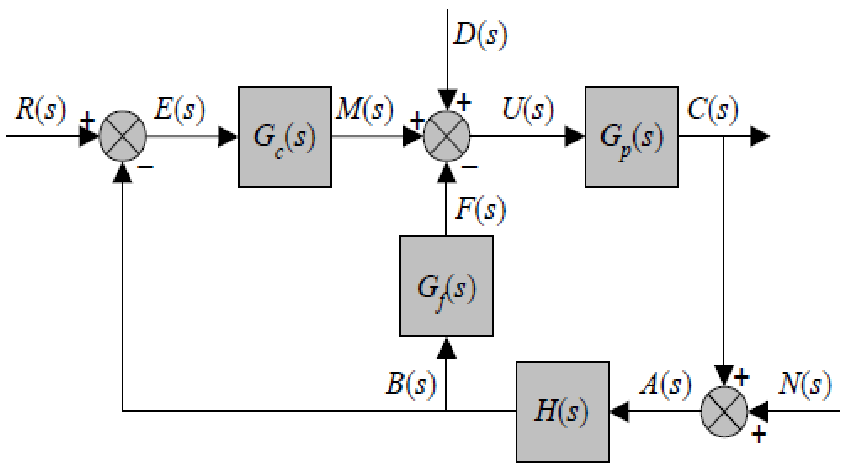
$N(s) = transferfucntion\ of\ the\ noise\ imput\ n(t)$

1. Background

In modeling system dynamics, ***transfer function*** approach is a value method in analyzing the response and obtaining computational solution. In the field of system dynamics, transfer functions used to characterize the input output relationships of components or systems that can be describe by linearly, time variant differential equations.

Block diagrams of dynamic systems is a pictorial representation of the functions performed by each component of the system and of the flow of signals within the system. Such a diagram depicts the interrelationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating the signal flows of the actual system more realistically.

2. Theory



Transfer functions:

$$E(s) = R(s) - B(s)$$

$$M(s) = E(s) * G_c(s)$$

$$U(s) = M(s) - F(s) + D(s)$$

$$C(s) = U(s) * G_p(s)$$

$$F(s) = B(s) * G_f(s)$$

$$A(s) = C(s) + N(s)$$

$$B(s) = A(s) * H(s)$$

3. Determination of the trasnfer functions:

$$a)\ \left|\frac{C(s)}{R(s)}\right|_{D(s)=N(s)=0}$$

$$E(s) = R(s) - B(s)$$

$$M(s) = E(s) * G_c(s)$$

$$U(s) = M(s) - F(s) \cancel{+ D(s)}$$

$$C(s) = U(s) * G_p(s)$$

$$F(s) = B(s) * G_f(s)$$

$$A(s) = C(s) \cancel{+ N(s)}$$

$$B(s) = A(s) * H(s)$$

$$C(s) = G_p(s)(M(s) - F(s))$$

$$C(s) = G_p(s)(E(s) * G_c(s) - B(s) * G_f(s))$$

$$C(s) = G_p(s)((R(s) - B(s)) * G_c(s) - B(s) * G_f(s))$$

$$C(s) = G_p(s) * G_c(s) * R(s) - G_p(s) * G_c(s) * B(s) - B(s) * G_f(s) * G_p(s)$$

$$C(s) = G_p(s) * G_c(s) * R(s) - B(s)(G_p(s) * G_c(s) + G_f(s) * G_p(s))$$

$$C(s) = G_p(s) * G_c(s) * R(s) - A(s) * H(s)(G_p(s) * G_c(s) + G_f(s) * G_p(s))$$

$$C(s) = G_p(s) * G_c(s) * R(s) - C(s) * H(s)(G_p(s) * G_c(s) + G_f(s) * G_p(s))$$

$$C(s) = G_p(s) * G_c(s) * R(s) - C(s) * H(s) * G_p(s) * G_c(s) - C(s) * H(s) * G_f(s) * G_p(s)$$

$$C(s) + C(s) * H(s) * G_p(s) * G_c(s) + C(s) * H(s) * G_f(s) * G_p(s) = G_p(s) * G_c(s) * R(s)$$

$$C(s)(1 + H(s) * G_p(s) * G_c(s) + H(s) * G_f(s) * G_p(s)) = G_p(s) * G_c(s) * R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G_p(s) * G_c(s)}{1 + H(s) * G_p(s) * G_c(s) + H(s) * G_f(s) * G_p(s)}$$

$$b)\ \left|\frac{C(s)}{D(s)}\right|_{N(s)=R(s)=0}$$

$$E(s) = \cancel{R(s)} - B(s)$$

$$M(s) = E(s) * G_c(s)$$

$$U(s) = M(s) - F(s) + D(s)$$

$$C(s) = U(s) * G_p(s)$$

$$F(s) = B(s) * G_f(s)$$

$$A(s) = C(s) \cancel{+ N(s)}$$

$$B(s) = A(s) * H(s)$$

$$C(s) = U(s) * G_p(s)$$

$$C(s) = G_p(s) * (\ M(s) - F(s) + D(s))$$

$$C(s) = G_p(s) * (\ E(s) * G_c(s) - B(s) * G_f(s) + D(s))$$

$$C(s) = G_p(s) * (-B(s) * G_c(s) - B(s) * G_f(s) + D(s))$$

$$C(s) = (G_p(s) * -B(s) * G_c(s) - G_p(s) * B(s) * G_f(s) + G_p(s) * D(s))$$

$$C(s) = -B(s)(G_p(s) * G_c(s) + G_p(s) * G_f(s)) + G_p(s) * D(s)$$

$$C(s) = -A(s) * H(s)(G_p(s) * G_c(s) + G_p(s) * G_f(s)) + G_p(s) * D(s)$$

$$C(s) = -C(s) * H(s)(G_p(s) * G_c(s) + G_p(s) * G_f(s)) + G_p(s) * D(s)$$

$$C(s) + C(s) * H(s) * G_p(s) * G_c(s) + C(s) * H(s) * G_p(s) * G_f(s) = G_p(s) * D(s)$$

$$C(s)(1 + H(s) * G_p(s) * G_c(s) + H(s) * G_p(s) * G_f(s)) = G_p(s) * D(s)$$

$$\frac{C(s)}{D(s)} = \frac{G_p(s)}{1 + H(s) * G_p(s) * G_c(s) + H(s) * G_p(s) * G_f(s)}$$

$$c)\ \left|\frac{C(s)}{N(s)}\right|_{R(s)=D(s)=0}$$

$$E(s) = \cancel{R(s)} - B(s)$$

$$M(s) = E(s) * G_c(s)$$

$$U(s) = M(s) - F(s) \cancel{+ D(s)}$$

$$C(s) = U(s) * G_p(s)$$

$$F(s) = B(s) * G_f(s)$$

$$A(s) = C(s) + N(s)$$

$$B(s) = A(s) * H(s)$$

$$C(s) = U(s) * G_p(s)$$

$$C(s) = G_p(s) * (\ M(s) - F(s))$$

$$C(s) = G_p(s) * (\ E(s) * G_c(s) - B(s) * G_f(s))$$

$$C(s) = G_p(s) * -B(s)(G_c(s) + G_f(s))$$

$$C(s) = G_p(s) * -A(s) * H(s) * (G_c(s) + G_f(s))$$

$$C(s) = G_p(s) * -(C(s) + N(s)) * H(s) * (G_c(s) + G_f(s))$$

$$C(s) = -H(s) * G_p(s) * (C(s) + N(s)) * (G_c(s) + G_f(s))$$

$$C(s) = -H(s) * G_p(s) * (C(s) * G_c(s) + C(s) * G_f(s) + N(s) * G_c(s) + N(s) * G_f(s))$$

$$C(s) + H(s) * G_p(s) * C(s) * G_c(s) + H(s) * G_p(s) * C(s) * G_f(s)$$

$$= -H(s) * G_p(s) * N(s) * G_c(s) - H(s) * G_p(s) * N(s) * G_f(s))$$

$$C(s)(1 + H(s) * G_p(s) * G_c(s) + H(s) * G_p(s) * G_f(s))$$

$$= N(s)(-H(s) * G_p(s) * G_c(s) + -H(s) * G_p(s) * G_f(s))$$

$$\frac{C(s)}{N(s)} = -\frac{H(s) * G_p(s) * G_c(s) + H(s) * G_p(s) * G_f(s)}{1 + H(s) * G_p(s) * G_c(s) + H(s) * G_p(s) * G_f(s)}$$

$$d)\ \left|\frac{E(s)}{R(s)}\right|_{D(s)=N(s)=0}$$

$$E(s) = R(s) - B(s)$$

$$M(s) = E(s) * G_c(s)$$

$$U(s) = M(s) - F(s) \sout{+ D(s)}$$

$$C(s) = U(s) * G_p(s)$$

$$F(s) = B(s) * G_f(s)$$

$$A(s) = C(s) \sout{+ N(s)}$$

$$B(s) = A(s) * H(s)$$

$$E(s) = R(s) - A(s) * H(s)$$

$$E(s) = R(s) - C(s) * H(s)$$

$$E(s) = R(s) - U(s) * G_p(s) * H(s)$$

$$E(s) = R(s) - (M(s) - F(s)) * G_p(s) * H(s)$$

$$E(s) = R(s) - (E(s) * G_c(s) - B(s) * G_f(s)) * G_p(s) * H(s)$$

$$E(s) = R(s) - (E(s) * G_c(s) + (E(s) - R(s)) * G_f(s)) * G_p(s) * H(s)$$

$$E(s) = R(s) - (E(s) * G_c(s) + E(s) * G_f(s) - R(s) * G_f(s)) * G_p(s) * H(s)$$

$$E(s) = R(s) - (E(s) * G_c(s) * G_p(s) * H(s) + E(s) * G_f(s) * G_p(s) * H(s)$$

$$-R(s) * G_f(s) * G_p(s) * H(s))$$

$$E(s) + E(s) * G_c(s) * G_p(s) * H(s) + E(s) * G_f(s) * G_p(s) * H(s) = R(s) -$$

$$+R(s) * G_f(s) * G_p(s) * H(s)$$

$$E(s)(1 + G_c(s) * G_p(s) * H(s) + G_f(s) * G_p(s) * H(s) = R(s)(1 -$$

$$* G_f(s) * G_p(s) * H(s))$$

$$\frac{E(s)}{R(s)} = \frac{1 + G_f(s) * G_p(s) * H(s)}{1 + G_c(s) * G_p(s) * H(s) + G_f(s) * G_p(s) * H(s)}$$

$e)\ \left|\frac{E(s)}{D(s)}\right|_{N(s)=R(s)=0}$

$E(s) = \sout{R(s)} - B(s)$

$M(s) = E(s) * G_c(s)$

$U(s) = M(s) - F(s) + D(s)$

$C(s) = U(s) * G_p(s)$

$F(s) = B(s) * G_f(s)$

$A(s) = C(s) \sout{+ N(s)}$

$B(s) = A(s) * H(s)$

$E(s) = -B(s)$

$E(s) = -A(s) * H(s)$

$E(s) = -C(s) * H(s)$

$E(s) = -G_p(s) * H(s)\big(M(s) - F(s) + D(s)\big)$

$E(s) = -G_p(s) * H(s)\left(E(s) * G_c(s) - B(s) * G_f(s) + D(s)\right)$

$E(s) = -G_p(s) * H(s)\left(E(s) * G_c(s) + E(s) * G_f(s) + D(s)\right)$

$$E(s) = \begin{pmatrix} -G_p(s) * H(s) * E(s) * G_c(s) - G_p(s) * H(s) * E(s) * G_f(s) \\ -G_p(s) * H(s) * D(s) \end{pmatrix}$$

$E(s) + G_p(s) * H(s) * E(s) * G_c(s) + G_p(s) * H(s) * E(s) * G_f(s) = -G_p(s) * H(s) * D(s)$

$E(s)(1 + G_p(s) * H(s) * G_c(s) + G_p(s) * H(s) * G_f(s)) = -G_p(s) * H(s) * D(s)$

$$\frac{E(s)}{D(s)} = -\frac{G_p(s) * H(s)}{1 + G_p(s) * H(s) * G_c(s) + G_p(s) * H(s) * G_f(s)}$$

$$f)\ \left|\frac{E(s)}{N(s)}\right|_{R(s)=D(s)=0}$$

$$E(s) = \cancel{R(s)} - B(s)$$

$$M(s) = E(s) * G_c(s)$$

$$U(s) = M(s) - F(s) \cancel{+ D(s)}$$

$$C(s) = U(s) * G_p(s)$$

$$F(s) = B(s) * G_f(s)$$

$$A(s) = C(s) + N(s)$$

$$B(s) = A(s) * H(s)$$

$$E(s) = -B(s)$$

$$E(s) = -(C(s) + N(s)) * H(s)$$

$$E(s) = -((M(s) - F(s)) * G_p(s) + N(s)) * H(s)$$

$$E(s) = -((E(s) * G_c(s) - B(s) * G_f(s)) * G_p(s) + N(s)) * H(s)$$

$$E(s) = -((E(s) * G_c(s) + E(s) * G_f(s)) * G_p(s) + N(s)) * H(s)$$

$$E(s) + ((E(s) * G_c(s) + E(s) * G_f(s)) * G_p(s) * H(s) = -N(s) * H(s)$$

$$E(s) + E(s) * G_c(s) * G_p(s) * H(s) + E(s) * G_f(s) * G_p(s) * H(s) = -N(s) * H(s)$$

$$E(s)\left(1 + G_c(s) * G_p(s) * H(s) + G_f(s) * G_p(s) * H(s)\right) = -N(s) * H(s)$$

$$\frac{E(s)}{N(s)} = -\frac{H(s)}{1 + G_c(s) * G_p(s) * H(s) + G_f(s) * G_p(s) * H(s)}$$

4. Find the steady state responses:

Determine the steady state response $c_{ss} = c(\infty) = \lim_{t\to\infty} c(t)$ and steady state error $e_{ss} = e(\infty) = \lim_{t\to\infty} e(t)$, That is,

- **Find $c_{ss}|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}}$ and $e_{ss}|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}}$ when the reference input r (t) is a step function: $\bar{R}.1(t)$ while the disturbance and noise inputs. d (t) and n (t), are zero.**

$$\rightarrow c_{ss}|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}} = c(\infty)|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}} = \lim_{t\to\infty} c(t)|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}}$$

$$\rightarrow \lim_{s\to 0} sC(s)|_{\substack{R(s)=\frac{\bar{R}.}{s}\\ D(s)=N(s)=0}} = \lim_{s\to 0} s\frac{C(s)}{R(s)}\Big|_{\substack{.\frac{\bar{R}.}{s}\\ D(s)=N(s)=0}}$$

$$\rightarrow \lim_{s\to 0} \cancel{s}\frac{G_p(s)*G_c(s)}{1+H(s)*G_p(s)*G_c(s)+H(s)*G_f(s)*G_p(s)}\frac{\bar{R}.}{\cancel{s}}$$

$$\rightarrow \frac{G_p(0)*G_c(0)\bar{R}}{1+H(0)*G_p(0)*G_c(0)+H(0)*G_f(0)*G_p(0)}$$

$$\rightarrow e_{ss}|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}} = e(\infty)|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}} = \lim_{t\to\infty} e(t)|_{\substack{r(t)=\bar{R}.1(t)\\ d(t)=n(t)=0}}$$

$$\rightarrow \lim_{s\to 0} sE(s)|_{\substack{R(s)=\frac{\bar{R}.}{s}\\ D(s)=N(s)=0}} = \lim_{s\to 0} s\frac{E(s)}{R(s)}\Big|_{\substack{.\frac{\bar{R}.}{s}\\ D(s)=N(s)=0}}$$

$$\rightarrow \lim_{s\to 0} \cancel{s}\frac{1+G_f(s)*G_p(s)*H(s)}{1+G_c(s)*G_p(s)*H(s)+G_f(s)*G_p(s)*H(s)}\frac{\bar{R}.}{\cancel{s}}$$

$$\rightarrow \frac{\left(1+G_f(0)*G_p(0)*H(0)\right)*\bar{R}}{1+G_c(0)*G_p(0)*H(0)+G_f(0)*G_p(0)*H(0)}$$

- **Find $c_{ss}\big|_{\substack{d(t)=\bar{D}1(t)\\ n(t)=r(t)=0}}$ and $e_{ss}\big|_{\substack{d(t)=\bar{D}.1(t)\\ n(t)=r(t)=0}}$ when the disturbance input d (t) is a step function: $\bar{D}.1(t)$ while the noise and reference inputs. n (t) and r(t), are zero.**

$$\rightarrow c_{ss}\big|_{\substack{d(t)=\bar{D}.1(t)\\ n(t)=r(t)=0}} = c(\infty)\big|_{\substack{d(t)=\bar{D}.1(t)\\ n(t)=r(t)=0}} = \lim_{t\to\infty} c(t)\big|_{\substack{d(t)=\bar{D}.1(t)\\ n(t)=r(t)=0}}$$

$$\rightarrow \lim_{s\to 0} sC(s)\big|_{\substack{D(s)=\frac{\bar{D}.}{s}\\ N(s)=R(s)=0}} = \lim_{s\to 0} s\frac{C(s)}{D(s)}\Big|_{\substack{\frac{\bar{D}.}{s}\\ N(s)=R(s)=0}}$$

$$\rightarrow \lim_{s\to 0} \cancel{s} \frac{G_p(s)}{1+H(s)*G_p(s)*G_c(s)+H(s)*G_p(s)*G_f(s)} \frac{\bar{D}.}{\cancel{s}}$$

$$\rightarrow \frac{G_p(0)*\bar{D}}{1+H(0)*G_p(0)*G_c(0)+H(0)*G_p(0)*G_f(0)}$$

$$\rightarrow e_{ss}\big|_{\substack{d(t)=\bar{D}.1(t)\\ n(t)=r(t)=0}} = e(\infty)\big|_{\substack{d(t)=\bar{D}.1(t)\\ n(t)=r(t)=0}} = \lim_{t\to\infty} e(t)\big|_{\substack{d(t)=\bar{D}.1(t)\\ n(t)=r(t)=0}}$$

$$\rightarrow \lim_{s\to 0} sE(s)\big|_{\substack{D(s)=\frac{\bar{D}.}{s}\\ N(s)=R(s)=0}} = \lim_{s\to 0} s\frac{E(s)}{D(s)}\Big|_{\substack{\frac{\bar{D}.}{s}\\ N(s)=R(s)=0}}$$

$$\rightarrow \lim_{s\to 0} \cancel{s} - \frac{G_p(s)*H(s)}{1+G_p(s)*H(s)*G_c(s)+G_p(s)*H(s)*G_f(s)} \frac{\bar{D}.}{\cancel{s}}$$

$$\rightarrow -\frac{G_p(0)*H(0)*\bar{D}}{1+G_p(0)*H(0)*G_c(0)+G_p(0)*H(0)*G_f(0)}$$

- **Find $c_{ss}\big|_{\substack{n(t)=\bar{N}.1(t)\\ r(t)=d(t)=0}}$ and $e_{ss}\big|_{\substack{n(t)=\bar{N}1(t)\\ r(t)=d(t)=0}}$ when the noise input n (t) is a step function: $\bar{N}.1(t)$ while the reference and disturbance inputs. r (t) and d (t), are zero.**

$$\rightarrow c_{ss}\big|_{\substack{n(t)=\bar{N}.1(t)\\ r(t)=d(t)=0}} = c(\infty)\big|_{\substack{n(t)=\bar{N}.1(t)\\ r(t)=d(t)=0}} = \lim_{t\to\infty} c(t)\big|_{\substack{n(t)=\bar{N}.1(t)\\ r(t)=d(t)=0}}$$

$$\rightarrow \lim_{s\to 0} sC(s)\big|_{\substack{N(s)=\frac{\bar{N}.}{s}\\ R(s)=D(s)=0}} = \lim_{s\to 0} s\frac{C(s)}{N(s)}\Big|_{\substack{\frac{\bar{N}.}{s}\\ R(s)=D(s)=0}}$$

$$\rightarrow \lim_{s\to 0} s - \frac{H(s)*G_p(s)*G_c(s)+H(s)*G_p(s)*G_f(s)}{1+H(s)*G_p(s)*G_c(s)+H(s)*G_p(s)*G_f(s)}\frac{\bar{N}.}{s}$$

$$= -\frac{\left(\boldsymbol{H(0)*G_p(0)*G_c(0)+H(0)*G_p(0)*G_f(0)}\right)\bar{N}}{\boldsymbol{1+H(0)*G_p(0)*G_c(0)+H(0)*G_p(0)*G_f(0)}}$$

$$\rightarrow e_{ss}\big|_{\substack{n(t)=\bar{N}.1(t)\\ r(t)=d(t)=0}} = e(\infty)\big|_{\substack{n(t)=\bar{N}.1(t)\\ r(t)=d(t)=0}} = \lim_{t\to\infty} e(t)\big|_{\substack{n(t)=\bar{N}.1(t)\\ r(t)=d(t)=0}}$$

$$\rightarrow \lim_{s\to 0} sE(s)\big|_{\substack{N(s)=\frac{\bar{N}.}{s}\\ R(s)=D(s)=0}} = \lim_{s\to 0} s\frac{E(s)}{N(s)}\Big|_{\substack{\frac{\bar{N}.}{s}\\ R(s)=D(s)=0}}$$

$$\rightarrow \lim_{s\to 0} s - \frac{H(s)}{1+G_c(s)*G_p(s)*H(s)+G_f(s)*G_p(s)*H(s)}\frac{\bar{N}.}{s}$$

$$= -\frac{\boldsymbol{H(0)*\bar{N}}}{\boldsymbol{1+G_c(0)*G_p(0)*H(0)+G_f(0)*G_p(0)*H(0)}}$$